MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2015/2016

TMA 1111 – MATHEMATICAL TECHNIQUES

(All sections / Groups)

10 OCTOBER 2015
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **6 pages** only including the cover page with **5 Questions** and an **Appendix**.

2. There are **2 Sections**:
   Section A (3 Questions): Attempt **ALL THREE(3)** questions.
   Section B (2 Questions): Attempt **ONE (1)** question **ONLY**.
   All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write your answers in the Answer Booklet provided, and **start each question on a new page**.

4. Show all steps.

5. Formulas are provided and attached in Appendix.

SECTION A

Answer ALL THREE (3) Questions.

Question 1

a. Let $p = \langle 6, -1 \rangle$ and $q = \langle 5,6 \rangle$, find the ***angle***, θ, between p and q. **[2 marks]**

b. Consider the points $P(1,4,4)$, $Q(1,1,1)$ and $R(4,3,1)$.

i. Find the ***vectors*** $u = \vec{PQ}$ and $v = \vec{PR}$. **[4 marks]**

ii. Find the ***dot product*** of $\vec{PQ}$ and $\vec{PR}$, that is $u \bullet v$. **[2 marks]**

c. Determine the ***equation*** of the plane passing through point (2, 5, 4) and perpendicular to vector $n = \langle 7, 9, 7 \rangle$. **[2 marks]**

Question 2

a. A ***first order differential equation*** (DE) is given below:

$$6y\frac{dy}{dx} + 12x = 6x^2$$

i. Show that the DE is ***separable***. **[1 mark]**

ii. Solve the DE using ***separable method***. **[3 marks]**

b. Find the **general solution** of the following ***homogeneous linear second order differential equation***.

$$y'' + 5y' + 4y = 0$$ **[4 marks]**

c. Determine whether the differential equation, $3x(y^2 - 1)dx + (4y^2 + x)dy = 0$, is ***exact***. **[2 marks]**

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Question 3

The manager of *IT solutions* is studying his company's profit. He would like to know the relationship between the amount of expenses and the charges of his projects. Data below is collected from his past projects.

Charge , y (RM, in thousands)	31.5	43.0	45.0	46.5	32.0	49.5	38.1	32.0
Expense , x (RM, in thousands)	13.9	17.6	17.0	16.5	13.6	18.2	15.5	14.8

a. Compute $\bar{x}, \bar{y}, S_{xy}, S_{xx}$ and S_{yy}. **[6 marks]**

b. Find the ***regression model***, $\hat{y} = \beta_0 + \beta_1 x$, to fit the data. **[1.5 marks]**

c. What will be the charge of a project if its expense is RM16000? **[1 mark]**

d. What is the value of ***correlation coefficient, r*** ? What does this value indicate? **[1.5 marks]**

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SECTION B

Answer ONE(1) Question ONLY.

Question 4

a. Solve the system of linear equations by using the ***inverse*** of a matrix.

$$\begin{aligned} x_1 - 3x_2 + x_3 &= 2 \\ x_1 + x_2 &= 6 \\ 2x_1 + x_3 &= 4 \end{aligned}$$

[7 marks]

b. Express $z = \sqrt{2} - \sqrt{2}\,i$ in ***polar*** and ***exponential*** forms. **[3 marks]**

Question 5

a. For each of the following limits, determine if it exists.

i. $\lim_{x\to\infty} \frac{8-x^5}{2x^6+7}$. [Hint: L'Hopital's rule may be useful.] **[2 marks]**

ii. $\lim_{x\to 3} (-2x^2 - 5x + 5)$. **[1 mark]**

b. i. Find y' of the function, $y = x^2(2+8x)^3$. [Hint: Product rule may be useful.] **[2 marks]**

ii. Find y'' of the function, $y = 3x^4 - e^x$. **[2 marks]**

c. Evaluate

i. $\int (x^4 - 0.25 + \frac{1}{x})dx$ **[1 mark]**

ii. $5\int_0^1 xe^x dx$. [Hint: Integration by part may be useful.] **[2 marks]**

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APPENDIX

1. Length of a vector v : $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}$

2. $\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos\theta$

3. Dot product: $\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$

4. Point-Normal Equation of a Plane: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

5. Criterion for an Exact Differential Equation : $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$

6. General solution of the homogeneous linear second order differential equation

   For Distinct real roots: $y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$

   For Repeated roots: $y = (c_1 + c_2 x) e^{\lambda x}$

   For Complex conjugate roots: $y = e^{\alpha x} (c_1 \cos \beta x + c_2 \sin \beta x)$

7. Derivative Formulas

$\dfrac{d}{dx} e^x = e^x$
Power Rule: $\dfrac{d}{dx} x^n = nx^{n-1}$ if $f(x) = x^n$ with $n \in R$
Product Rule: $\dfrac{d}{dx}(f \cdot g) = f \dfrac{dg}{dx} + g \dfrac{df}{dx}$

8. Integration Formulas

$\int x^n dx = \dfrac{x^{n+1}}{n+1} + C$, for $n \neq -1$, n rational
$\int \dfrac{1}{x} dx = \ln x + C$
$\int e^x dx = e^x + C$

9. $\bar{x} = \dfrac{\sum x}{n}$

Continued.....

10. $S_{xy} = \sum xy - \dfrac{\left(\sum x \sum y\right)}{n}, \qquad S_{xx} = \sum x^2 - \dfrac{\left(\sum x\right)^2}{n}, \qquad S_{yy} = \sum y^2 - \dfrac{\left(\sum y\right)^2}{n}$

11. $\beta_1 = \dfrac{S_{XY}}{S_{XX}}, \qquad \beta_0 = \bar{y} - \beta_1 \bar{x}$

12. Sample correlation coefficient = $r = \dfrac{S_{xy}}{\sqrt{S_{xx}}\sqrt{S_{yy}}}$

13. $|z| = \sqrt{x^2 + y^2}$

14. Polar form: $z = r(\cos\theta + i\sin\theta)$, Exponential form: $z = re^{i\theta}$

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